responding degrees of freedom of the same or of other particles. For instance, Maxwell's Theorem itself is not proved if the spheres of one system have not as perfect freedom for mutual collisions as for collisions with these of the other system. We are not entitled to *assume* that they can then acquire, much less that they can maintain, the "special" state on which the further argument is based. This is one of the traps into which Clerk-Maxwell fell; for he assumed that the result could be deduced from the consequences of a sort of typical impact between two particles, one from each system, moving in directions at right angles to one another, and each having the mean-square speed of its own system.

Let the masses of two impinging spheres, whose coefficient of restitution is unity, be P and Q; and let u and v, measured towards the same parts, be the components of their velocities along the line of centres at impact. Let these become, after impact, u' and v'. Then we have, as in the text-books,

$$P(u' - u) = -\frac{2PQ}{P+Q}(u - v) = -Q(v' - v);$$

which gives, at once,

$$P(u'^2 - u^2) = -\frac{4PQ}{(P+Q)^2}(Pu^2 - Qv^2 - (P - Q)uv)$$
$$= -Q(v'^2 - v^2).$$

Each of these equal quantities is double the amount of energy transferred from one sphere to the other.

Now, when kinetic equilibrium has been (at least approximately) arrived at, such transference must (on the average) cease:—so that the equilibrium condition will be

$$P\overline{u^2} - Q\overline{v^2} - (P - Q)\overline{uv} = 0,$$

where the bars indicate average values.

Everything turns on the proper estimation of these averages. For, if the average of uv be taken as zero, we have Clerk-Maxwell's result; provided that $P\overline{u^2}$ and $Q\overline{v^2}$ be proportional to the average energy of a P and a Q respectively. This is a comparatively obvious trap.

But if we consider that collisions are more likely to occur between two particles, having *given* speeds, if they be moving towards opposite parts than if towards the same parts, we see that, on the average, u and v are more likely to have unlike, than like, signs; and therefore that the value of $\overline{uv}$ is negative. It is not so easy to see, beforehand, what sort of changes this consideration may produce in the values of $\overline{u^2}$ and $\overline{v^2}$.

This leads to an inquiry as to the relation between the relative speed of two particles and the probability of their collision, and the formulæ become complicated.

I found, by an *approximate* investigation in which the above consideration was given effect to, that, if the average energies of a P and a Q be called, as usual, $3P\alpha^2/2$ and $3Q\beta^2/2$, we have, nearly,

$$\overline{u^2} = \alpha^2/2, \quad \overline{v^2} = \beta^2/2; \quad \overline{uv} = -e\alpha\beta;$$

where e depends only on the relative magnitudes of α and β. If this were true, it would follow at once that the average energy per sphere would be less for those of greater mass.[1]

But I soon found that at least part of this must be erroneous, because though many of its consequences would require a mere modification of the *mode of stating* certain well-known theorems, others were incompatible with physical principles.

Yet it seemed (and this is a specially good instance of the pitfalls I have alluded to) hardly possible that, as $\overline{uv}$ is certainly negative, we could get $P\alpha^2 - Q\beta^2 = 0$ for the form of the above expression, except when $P = Q$.

When I revised my calculations, dispensing with methods of approximation, I found that, strange as it appears, the average value of $u(u - v)$, the P part of the above expression, depends on α only, and *does not involve* β! Its value is $2\alpha^2/3$, of which

$$\overline{u^2} = \frac{4\alpha^2 + 3\beta^2}{6(\alpha^2 + \beta^2)}\alpha^2 \text{ and } -\overline{uv} = \frac{\alpha^2\beta^2}{6(\alpha^2 + \beta^2)}.$$

If the above result, which has been obtained by the evaluation of a number of troublesome definite integrals, be correct, there must be some very direct and simple proof that $\overline{u(u-v)}$ depends on α only.

[1] This conclusion, after I had seen it to be erroneous, and had taken timely precautions, sufficient (as I thought) to prevent its appearing in NATURE, was unfortunately published as a definitely-ascertained fact; without any allusion to the *approximation* on which I had stated it to be based.

REPORT TO THE TRINITY HOUSE ON THE INQUIRY INTO THE RELATIVE MERITS OF ELECTRICITY, GAS, AND OIL AS LIGHTHOUSE ILLUMINANTS

THE Committee appointed by the Trinity House to report on the merits of electricity, gas, and mineral oil as lighthouse illuminants have recently issued a valuable report giving an account of the investigations carried out under their directions, and the conclusions they have arrived at. The Committee consisted of Elder Brethren of the Trinity House. They were assisted by Mr. A. Vernon Harcourt, who was appointed by the Board of Trade to co-operate with the Committee, and by Prof. W. Grylls Adams and Mr. Harold Dixon, in the more purely scientific part of their investigation.

Three temporary lighthouses were erected on the South Foreland, and fitted up for electricity, gas, and mineral oil; the optical arrangements were "multiform" in all three—that is, consisted of several similar sources of light, each with its own condensing lenses, superposed; in the case of the electrical tower there were three superposed lamps, as was also the case with the oil tower: but in the gas tower there were four lamps; the two former were therefore "triform," whereas the latter was a "quadriform" light." Any one lamp in either tower could be lighted independently of the others, so as, for instance, to permit biform electricity to be compared with triform oil and quadriform gas.

The lamps for the electric light, and the magneto-electric machines for working them, were supplied by M. de Meritens; the gas apparatus was that of Mr. Wigham, each burner consisting of 108 jets in concentric rings, of which a part only might be employed; the oil lamps in the third tower during the greater part of the trials were six-wicked Douglass pattern, but burners of this description with seven and eight concentric wicks were also tried at various times during the progress of the experiments.

In addition to the temporary lighthouses, three observing huts and a photometric gallery 380 feet long were erected.

The actual observations that were made may be divided into two classes—eye-estimations, and photometric measurements. The former were made by the Elder Brethren, by officers on board the light-vessels in the neighbourhood, by merchant officers in passing ships, and by the coastguard officers at those stations from which the lights were visible. These eye-observations were of two kinds:—(1) Estimations of the comparative brilliancy of the lights; (2) definite statements as to the various distances at which the lights were visible in hazy or foggy weather.

With reference to observations of the first kind, they were conducted in accordance with regulations issued by the Trinity House Committee: the observers were instructed in filling in the books of forms which were issued to them, to put down in one column the light from the

electrical tower as 100 and in the other column the estimated brilliancy of the lights exhibited by the other two towers as compared with it. It seems probable that the recorded numerical values of the relative brilliancy of the lights can only be a very rough approximation, and that the figures can hardly be taken as indicating with any degree of precision how much brighter one or other of the lights was on any particular occasion. This would probably be admitted by all who have any acquaintance with actual photometric measurements, and who therefore know how difficult it is to form any reliable judgment of the relative illumination of two surfaces, even when these surfaces are actually in contact, excepting the relation of equality. In the case of the experimental lights the comparison must have been rendered still more difficult by the fact that what was to be compared was not the comparative illumination of two moderately bright surfaces in close proximity, but the comparative brilliancy of two lights at some distance from each other, their very brightness adding to the difficulty.

Still these estimations are manifestly valuable as setting forth in a clear and unmistakable form that, to the average observer, a particular light appeared the most brilliant; and such seems to have been the way in which they were regarded by the Committee, for on page 21 they state "it will be evident that by mere eye-measurement proportions can only be approximately determined, although the order of superiority may be accepted as proved."

The results of these determinations are set forth in four tables, from which it appears that in clear weather, and in weather that, although not absolutely clear, was not very foggy, there was no question as to the absolute superiority of the electric light over both its competitors, the electric light in the single form having a superiority of more than 30 per cent. assigned to it, as over gas, or oil, in their highest powers (*i.e.* quadriform for gas, and triform for oil); the large-sized gas-burner, with 108 jets, appears to have been slightly superior to the six-wick oil-burner, and consequently the quadriform gas to the triform oil.

The eye-observations of the second kind, those in which the distances at which the lights were visible in foggy weather were recorded, gave much the same result: that the electric light penetrated through the fog to the greatest distance, and that the oil and the gas were about equal in their penetrating power.

These observations also showed that in the case of the electricity the best result was obtained when the currents produced by two or even three machines were sent through a single lamp, and not when each of the lamps was worked by its own special current.

The photometric measurements were carried out by Mr. Dixon, Mr. Harcourt's pentane flame being used as the standard. As is well known, Mr. Harcourt's standard is an air-gas flame which, unlike the so-called standard candles still commonly used for photometric purposes, is not subject to irregular variations in its light-producing powers. Part II. of the Report contains a full account of the standard flame, and the two arrangements for producing it, both of which were in use at the South Foreland. In Mr. Harcourt's original arrangement the air-gas was made and stored in a gas-holder by causing a volume of pentane to diffuse into a known volume of air, and then burning the mixture under certain definite conditions which could be accurately produced at all times. The conditions were such that the flame emitted the same amount of light as an average sperm candle burning under the conditions laid down in the Acts of Parliament which control the quality of the metropolitan gas supply, an amount of light which may differ considerably from that emitted by any single candle.

Mr. Harcourt's pentane lamp was also used; in this arrangement the air-gas is produced as it is required. The lamp is very simple in construction, and the flame is just as constant as in the older form, and as easily regulated, whilst, unlike the older form, the lamp is extremely portable, the whole apparatus not occupying much more space than a packet of candles.

Two kinds of photometer were used: a bar-photometer with a Leeson star disk, and Mr. Harcourt's table-photometer. The latter is a variety of shadow-photometer, and possesses two special advantages:—(1) In common with all shadow-photometers the two sources of light are on the same side of the illuminated surface, and therefore there is less risk of the results being rendered untrustworthy by diffused or accidentally reflected light than when, as in the more commonly employed arrangements, the sources of light are on opposite sides. (2) The comparison being made by altering the size of the flames, and not their distance, the two portions of the illuminated surface do not alter their relative position, and are always in that which is most favourable for comparison, accurate juxtaposition. The difference in colour between the arc light and the pentane rendered it impossible to employ the shadow-photometer for the estimation of the electric light. For these measurements a Leeson star disk was employed, and it was found that reliable measurements could be obtained by placing the disk between the two lights and moving it to and fro until the pattern of the star was equally distinct on either side, although on the two sides the colours of the pattern and the background were reversed.

There was so little difference between the colour of the gas and oil flames and that of the pentane flame, that in the case of these two illuminants measurements could be made both with the star disk and with the shadow-photometer.

Comparisons were made in the photometric shed of the light emitted by the De Meritens electric lamp; the Wigham gas-burners with different numbers of jets up to the maximum of 108; the Douglass Argand gas-burner; the Sugg gas-burner; and the Siemens regenerative gas-burner; and also the six- and seven-wick Douglass oil-burners.

The amount of light emitted by each of the experimental lighthouses was also determined, the observations being made in the huts which had been erected for this purpose at different distances from the towers. At the hut nearest to the towers the light from all the burners could be compared directly with the pentane lamp giving the light of one candle, but at the second hut only the electric light and the higher powers of the gas and oil lights could be directly compared with the pentane lamp; the single gas and oil lights had to be condensed by a lens before accurate measurements of them could be taken; an achromatic lens, lent by the Astronomer-Royal, was used for this purpose. The fraction of the light lost by the absorption and reflection of the lens was experimentally determined and allowed for in the observation.

The general result of a very large number of observations appears to have been that there is but little to choose between oil and gas as far as their illuminating powers are concerned, and that electricity is greatly superior to both.

The experiments brought out one fact of great practical as well as scientific interest—that remarkable changes in the transparency of the air occur without any visible haze or mist. To quote Mr. Dixon's words:—"Invisible clouds seemed to float by, obscuring the lights for a time as they passed across our line of vision. Sometimes the French lights at Calais and Cape Grisnez showed brilliantly, when the photometer at Hut 2 proved that the lights from the experimental towers, only a mile and a quarter away, had lost one-fourth to one-third their power."

With a view of further investigating the fog-penetrating powers of these different lights, the photometer shed was

filled with an artificial steam-fog, by means of a pipe brought from the boiler of the engine-house, and the 108-jet Wigham gas-burner, and an electric arc fed by one machine, were pitted against each other, and the distances from which the lights could be seen determined. In all cases the electric arc became visible before the gas flame, as the observers walked up the shed towards the lights, confirming the other eye-observations which have been already mentioned.

The experiments showed also that the electric light suffered a greater proportional loss than either of the two other illuminants when passing through fog or haze, but that, owing to its far greater initial intensity, it nevertheless exceeded the other lights in its penetrating power.

The Committee add to their Report some account of the cost of the three illuminants, from which it appears that there is but little difference in the first cost of the electric and gas systems, the latter being slightly the more costly; but, on the other hand, the annual cost of the gas is estimated at rather less than that of the electricity. The cost of the mineral-oil apparatus is estimated, both for its installation and for its annual maintenance, at about two-thirds that of either gas or electricity.

The general conclusions arrived at by the Committee—conclusions which seem fully borne out by the evidence set forth in the Report—are, that the "electric light, as exhibited in the experimental tower at South Foreland, has proved to be the most powerful light under all conditions of weather, and to have the greatest penetrative power in fog;" that for all practical purposes the gas and oil were equal; and "that for the ordinary necessities of lighthouse illumination, mineral oil is the most suitable and economical illuminant, and that for salient headlands, important landfalls, and places where a very powerful light is required, electricity offers the greatest advantages."

GEOGRAPHICAL EDUCATION

THE Council of the Royal Geographical Society have been making a determined effort during the past eighteen months to improve the position of geography in the education of this country, with special reference to the higher schools and universities. They have collected information as to the position of the subject and the methods used in teaching it in the schools and universities of the leading Continental countries as well as in England, and published the results in the form of a Report, which has attracted considerable attention, and is likely to lead to useful results. In connection with this inquiry the Society has arranged an Exhibition of Appliances in use in Geographical Education at the rooms, 53, Great Marlborough Street, which, since it was opened in the beginning of December, has attracted many visitors of the class for whose special benefit it was intended. Already there are signs that this Exhibition will do real good in at least leading to the multiplication and improvement of the meagre appliances in use in English schools. Nothing could show more strikingly the marked difference in the variety and quality of these appliances in use in English and in Continental schools.

The catalogue of the Exhibition covers 80 large octavo pages. It is arranged in eight divisions—wall-maps, globes, telluria, planetaria, &c., models and relief-maps, geographical and ethnological pictures, atlases, text-books, miscellaneous. Upwards of 200 wall-maps are shown, while about 100 more have not been hung for want of space. All the leading types of this kind of work are represented on the walls. They are arranged geographically—maps of the world, of Europe, and parts of Europe, and so on. The object aimed at in hanging the maps has been to bring side by side those of the same region by different publishers and in use in different countries, so that visitors may compare the results for themselves. Some of the English work thus shown is certainly good—accurate, carefully executed, and fairly well adapted for its purpose, especially the maps of Stanford and Keith Johnston. The Exhibition, it should be remembered, is purely educational, and therefore the finest cartographical work of our best publishers and our Ordnance Survey must not be looked for; some of this work will compare favourably with the best work of other countries. As a whole, it must be admitted, that English school-maps are far behind those of the leading Continental countries, notably Germany, Austria, Switzerland, and even Italy. We do not seem to be guided by any definite principles in the construction of such maps; our teachers, as a rule, have never seen good school-maps, and the best English cartographers seem to think such work beneath them. We in England seem to cherish the pernicious idea that a school-map should be something quite different in kind from an ordinary map of the best class. In the Continental countries mentioned above, on the other hand, it is recognised that in the case of young people, even more than with men and women, only the very best work should be used, for first impressions are everything. In elementary wall-maps, of course, the minute details of the finest hill-shading and other features would be out of place, but the style and method of the work should be the same, only more generalised. For example, in Austrian schools, maps produced by photography from reliefs are absolutely forbidden on account of the exaggerated impression which they convey. The reliefs are almost necessarily exaggerated in such cases, and the light thrown on them from a particular direction to give picturesque effect; the result as a rule being a misrepresentation of the real configuration of the ground. Maps which attempt to indicate physical features by the use of variety of colour are but little used on the Continent. They do not appeal at all to the eye or help the imagination of the child, and are of no use in helping him to read maps executed in the usual way, which are the maps he must deal with when he grows up to be a man. The use of colour for special features is certainly useful, but then only in advanced classes. For the younger classes in Continental schools one does find it, but almost invariably conjoined with graphic mountain-shading. In the best maps, moreover, when the method is used, often only one, generally not more than two, colours are introduced: green for the lowest levels, tints of brown for the higher levels. In Kiepert's maps, brown alone, in deepening tints according to altitude, is used, just as blue is used after a similar fashion to indicate the varying depths of the sea. And this reminds us of the common practice in the best Continental schools, of always having two maps of the same region for teaching-purposes—one physical and the other political. On the former, always without names and political indications, the physical features are everything, and are boldly brought out; in the latter the physical features are still clear and prominent, but are accompanied by what are known as "political" features. In England one map has usually to serve not only for all grades of classes, but for both physical and political teaching: and as in our best school-maps the physical features are faint and obscured by the glaring colours used for political divisions, they are almost hidden when covered with names and other details.

In the matter of outward appearance, even, our school-maps leave room for great improvement. They, as a whole, cannot be compared as to taste and style with the best Continental maps. The finest of these are either not coloured at all, or the colours are put on faintly and delicately to show political divisions; often only boundary lines are coloured, so that the physical features, which have so much to do with political development, are well shown. The taste of our teachers and map-makers in this matter requires radical reformation; the more glaring and vivid the colouring of wall-maps, the more